CHAOS CONTROL FOR TWO CHEMICAL SYSTEMS

CONTROLUL HAOSULUI A DOUA SISTEME CHIMICE

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Abstract. *Over the last years, there has been considerable progress in generalizing the concept of synchronization to include the case of coupled chaotic oscillator's especially technical reasons. When the complete synchronization is achieved, the states of both systems become practically identical, while their dynamics in time remains chaotic. Many examples of synchronization have been documented in the literature, but currently theoretical understanding of the phenomena lags behind experimental studies. The main aim of this paper is to study the synchronization of two chemical chaotic systems (described by Willamowski–Rössler model) based on the adaptive feedback method of control. The transient time until synchronization depends on initial conditions of two systems and on the control strength. Then we can control these chemical chaotic systems in accordance with recent debates of Wang and Chen about full global synchronization and partial synchronization in a system of two or three coupled chemical chaotic oscillators.*
Key words: chemical reactions, synchronization, chaos control

Rezumat. *In ultimii ani s-a manifestat un progres considerabil în generalizarea conceptului de sincronizare pentru a include cazurile de oscilatori cuplați, în special din motive tehnice. Cǎnd este atinsă sincronizarea, starea celor două sisteme devine identică, deşi dinamica lor în timp rămâne haotică. In literatură s-au prezentat multe exemple de sincronizare in general teoretic, studii experimentale lipsind însă. Scopul principal al acestui articol este de a studia sincronizarea a două sisteme chimice haotice (descrise de modelul Willamowski–Rössler) pe baza unei metode de control de tip feedback. Timpul de tranziție până la sincronizare depinde de condițiile inițiale ale celor două sisteme şi de intensitatea controlorului. Deci putem controla aceste sisteme haotice chimice în acord cu recentele dezbateri ale lui Wang şi Chen legate de sincronizarea globală şi sincronizarea parțială a unui sistem cu doi sau 3 oscilatori chimici cuplați.*
Cuvinte cheie: reacții chimice, sincronizare, controlul haosului

INTRODUCTION

Over the last years, there has been considerable progress in generalizing the concept of synchronization to include the case of coupled chaotic oscillators' especially technical reasons. When the complete synchronization is achieved, the states of both systems become practically identical, while their dynamics in time remains chaotic. Many examples of synchronization have been documented in the literature, but currently theoretical understanding of the phenomena lags behind experimental studies (Chen et Dong, 1998; Grosu, 1997, Grosu et al., 2008; Lerescu et al.,

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2004, 2006; Oancea et al., 2011). The main aim of this paper is to study the synchronization of two chemical chaotic systems based on the adaptive feedback method of control. One of famous ideal chemical models is the Willamowski–Rössler model, which was proposed by Willamowski, Rössler and co-workers (Lei et al., 2005, 2009; Xu et. al., 2008). The Willamowski–Rössler model represents some chemical reactions and its mechanism consists in the following elementary steps:

$$A_1 + X \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} 2X,$$

$$X + Y \underset{k_{-2}}{\overset{k_2}{\rightleftharpoons}} 2Y,$$

$$A_5 + Y \underset{k_{-3}}{\overset{k_3}{\rightleftharpoons}} A_2,$$

$$X + Z \underset{k_{-4}}{\overset{k_4}{\rightleftharpoons}} A_3,$$

$$A_4 + Z \underset{k_{-5}}{\overset{k_5}{\rightleftharpoons}} 2Z.$$

THEORY

The nondimensionalized chemical dynamical evolution equations of the Willamowski - Rössler system are given as follows:

$$\begin{aligned} \dot{x}_1 &= -a_1 x_1 - k_{-1} x_1^2 - x_1 x_2 - x_1 x_3 \\ \dot{x}_2 &= x_1 x_2 - a_5 x_2 \\ \dot{x}_3 &= a_4 - x_1 x_3 - k_{-5} x_3^2 \end{aligned} \qquad (1)$$

with: $a_1=30$, $a_4=16.5$, $a_5=10$, $k_{-5}=0.5$ (2)

If k_1 is as the control parameter, system (1) can exhibit chaotic attractor when the value of k_1 is selected as 0.5.

Figure 1 shows that the attractor projected onto $x_1x_2x_3$ space for the chaotic system (1) with values from (2) and initial conditions $x_1(0)=1$, $x_2(0)=1$, $x_3(0)=1$

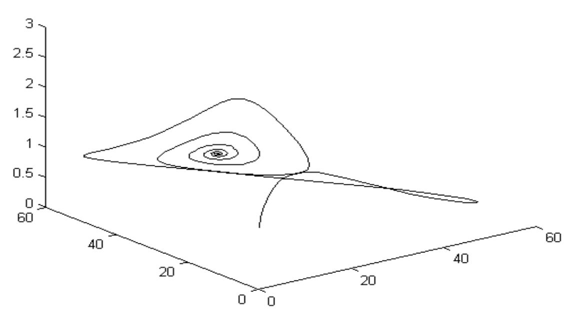

Fig. 1 – Phase portrait of (x_1, x_2, x_3) for Willamowski–Rössler system

To synchronize two Willamowski - Rössler systems we used a simple method for chaos synchronization proposed in (Guo et Li, 2007; Guo et al., 2009).

If the chaotic system (master) is:

$\dot{x} = f(x)$ where $x = (x_1, x_2, ...x_n) \in R_n$

$f(x) = (f_1(x), f_2(x).....f_n(x)) : R^n \rightarrow R^n$

The slave system is:

$\dot{y} = f(y) + \varepsilon(y - x)$

where the functions $\dot{\varepsilon}_i = -\lambda_i (y_i - x_i)^2$ and λ_i are positive constants

RESULTS AND DISCUSSION

The slave system for the system (1) is:

$$\dot{y}_1 = -30y_1 - 0.5y_1^2 - y_1 y_2 - y_1 y_3 + z_1(y_1 - x_1)$$
$$\dot{y}_2 = y_1 y_2 - 10y_2 + z_2(y_2 - x_2)$$
$$\dot{x}_3 = 16.5 - y_1 y_3 + z_3(y_3 - x_3) \quad (3)$$

The control strength is of the form:

$$\dot{z}_1 = -(y_1 - x_1)^2$$
$$\dot{z}_2 = -(y_2 - x_2)^2$$
$$\dot{z}_3 = -(y_3 - x_3)^2 \quad (4)$$

Fig. 2, 3, 4, 5 and 6 show the synchronization of the two systems

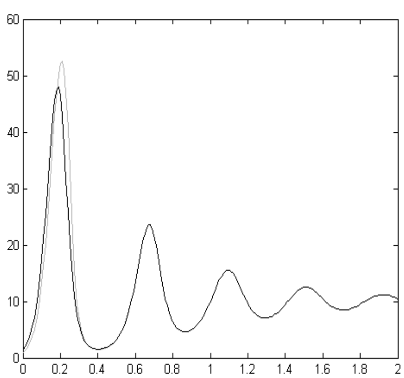

Fig.2 – $x_1(t)$- gray; $y_1(t)$- black [$x_1(0)$=1, $x_2(0)$=1, $x_3(0)$=1; $y_1(0)$ =1.5; $y_2(0)$ = 1.5 $y_3(0)$ = 1.5; $z_1(0)$ =1; $z_2(0)$ = 1 ; $z_3(0)$ =1]

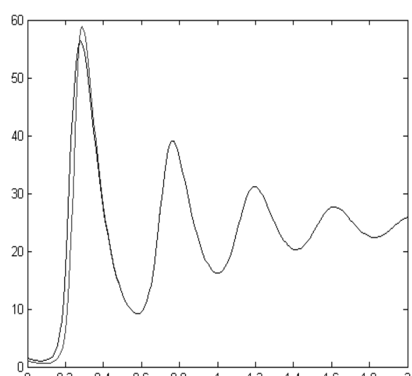

Fig.3 – $x_2(t)$- gray; $y_2(t)$- black [$x_1(0)$=1, $x_2(0)$=1, $x_3(0)$=1; $y_1(0)$ =1.5; $y_2(0)$ = 1.5 $y_3(0)$ = 1.5; $z_1(0)$ =1; $z_2(0)$ = 1 ; $z_3(0)$ =1]

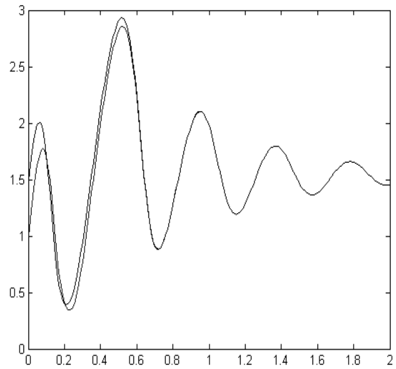

Fig. 4 – $x_3(t)$- gray; $y_3(t)$- black [$x_1(0)=1$, $x_2(0)=1$, $x_3(0)=1$; $y_1(0)=1.5$; $y_2(0) = 1.5$ $y_3(0) = 1.5$; $z_1(0)=1$; $z_2(0) = 1$; $z_3(0)=1$]

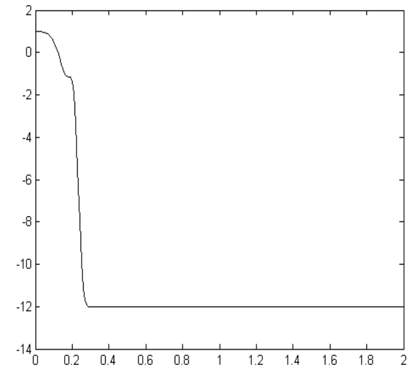

Fig. 5 – The control strength z_1[$x_1(0)=1$, $x_2(0)=1$, $x_3(0)=1$; $y_1(0)=1.5$; $y_2(0) = 1.5$ $y_3(0) = 1.5$; $z_1(0)=1$; $z_2(0) = 1$; $z_3(0)=1$]

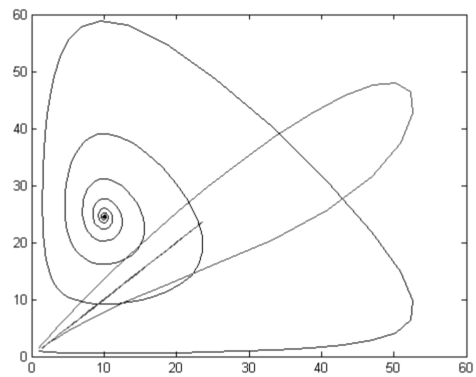

Fig. 6 – Phase portrait portrait of (x, x_2,) and (x,y_1) for WR system [$x_1(0)=1$, $x_2(0)=1$, $x_3(0)=1$; $y_1(0)=1.5$; $y_2(0) = 1.5$ $y_3(0) = 1.5$; $z_1(0)=1$; $z_2(0) = 1$; $z_3(0)=1$]

Debin Huang (2005), by testing the chaotic systems including the Lorenz system, Rossler system, Chua's circuit, and the Sprott's collection of the simplest chaotic flows found that we can use a single controller to achieve identical synchronization of a three-dimensional system (for Lorenz system this is possible only we add the controller in the second equation).

For the system (1), we achieved the synchronization if one controller is applied; the synchronization is faster if the controller is applied in the first or the second equation than the controller is applied in the third equation of the system. When all controllers are applied, the synchronization is faster than one controller is applied in every equation.

Some modifications of chaotic chemical Willamowski-Rössler system give special behavior of this system.

So, if we consider the modified system when variable x_2 and x_3 are inversed we obtain the modified Willamowski-Rössler system the the form:

$$\dot{x}_1 = -30x_1 - 0.5x_1^2 - x_1x_2 - x_1x_3 \quad (5)$$
$$\dot{x}_2 = x_1x_3 - 10x_2$$
$$\dot{x}_3 = 16.5 - x_1x_2 - 0.5x_2^2$$

and we obtain the attractor from figure 7:

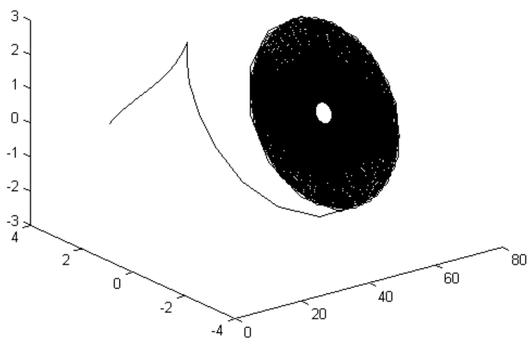

Fig. 7 – Phase portrait of (x_1, x_2, x_3) for modified Willamowski–Rössler system

The synchronization of the master with the slave system is given in Fig. 8 and 9:

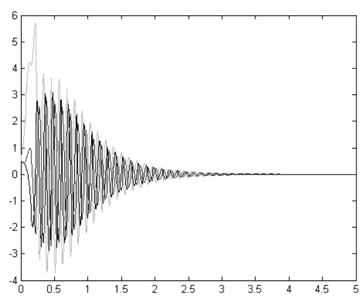

Fig. 8 – Synchronization errors between master and slave systems [$x_1(0)$=1, $x_2(0)$=1, $x_3(0)$=1; $y_1(0)$ =1.5; $y_2(0)$ = 1.5 $y_3(0)$ = 1.5; $z_1(0)$ =1; $z_2(0)$ = 1 ; $z_3(0)$ =1]

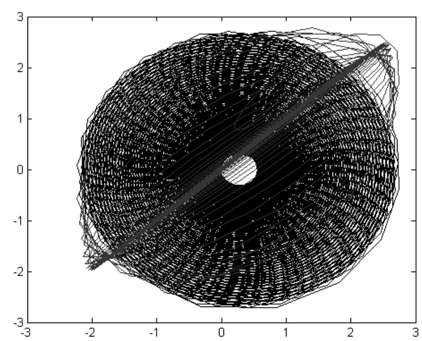

Fig. 9 – Phase portrait of (x_2 x_3,) and (x_2,y_3) for WR system [$x_1(0)$=1, $x_2(0)$=1, $x_3(0)$=1; $y_1(0)$ =1.5; $y_2(0)$ = 1.5 $y_3(0)$ = 1.5; $z_1(0)$ =1; $z_2(0)$ = 1 ; $z_3(0)$ =1]

CONCLUSIONS

In order to formulate the chaos control, the synchronization of two chaotic chemical systems (described by Willamowski–Rössler model) based on the adaptive feedback method of control is presented in this work. The transient time until synchronization depends on initial conditions of two systems and on the control strength. Therefore, we can control these chemical chaotic systems in accordance with recent debates of Wang and Chen (2010) about full global synchronization and partial synchronization in a system of two or three coupled chemical chaotic oscillators.

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